NOTATION

a = radius of sphere
c = specific heat
m = mass of sphere
m_f = mass of displaced fluid
p = Laplace transform parameter
Q = rate of heat generation in the sphere
t = time
T = temperature
T_0 = initial temperature
u = sphere velocity
V = dimensionless velocity of sphere, $V = u/[\beta$ (Stokes velocity)]

Greek Letters

α = thermal diffusivity
β = density ratio parameter, $9\rho_f/[2(\rho + \rho_f/2)]$
γ = density and specific heat ratio parameter, $\gamma = 3(\rho_f c_f)/(\rho c)$
θ = dimensionless temperature, $\theta = (T - T_0)/T_0$
$\lambda_1, \lambda_2, \lambda_3$ = heat transfer parameters, $\lambda_1 = ha/k_f$, $\lambda_2 = \frac{4\pi a^3 \rho_f c_f}{mc}\frac{ha}{k_f}$, $\lambda_3 = \frac{4\pi a^3 \rho_f c_f}{mc}$
ν = kinematic viscosity
ρ = density
τ = dimensionless time, $\tau = \nu t/a^2$ (fluid flow problem) and $\tau = \alpha_f t/a^2$ (heat transfer problem)

Subscripts and Superscripts

f = outer flow quantities
— = Laplace transformation

LITERATURE CITED

1. Sy, F., J. W. Taunton, and E. N. Lightfoot, *AIChE J.*, **16**, 386 (1970).
2. Basset, A. B., "Hydrodynamics," Dover, New York (1961).
3. Carslaw, H. S., and J. C. Jaeger, "Conduction of Heat in Solids," Oxford University Press, Oxford (1959).

Axial Dispersion of a Non-Newtonian Liquid in a Packed Bed

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Fluid behavior in a packed bed may be closely approximated by the socalled dispersion model. The degree of fluid mixing based on this model may be expressed in terms of Peclet number defined as $\bar{u}d_p/E$. Recently a correlation between N_{Pe} and N_{Re} for Newtonian fluid through packed bed was obtained by Chung and Wen (*2, 3*) as

$$\epsilon N_{Pe} = 0.2 + 0.011\ (N_{Re})^{0.48}$$

The dispersion coefficients can be conveniently obtained by a pulse-response technique. Hougen and Walsh (*6*) presented an extensive discussion of theoretical and practical aspects for the analysis of pulse data. To obtain the model parameters from the data of pulse testing, Johnson (*7*) used: (1) frequency response analysis, (2) the moment method analysis, and (3) the s-plane analysis. Clements and Schnelle (*4*) discussed the normalized frequency contents of various shapes of pulses. They showed that sharp pulses are needed to obtain high frequency content in the signal. Justice (*8*) used pulse techniques to study longitudinal dispersion in a packed bed with and without mass transfer.

EXPERIMENT

A schematic diagram of the apparatus used is shown in Figure 1. Water and 0.1 and 0.35% aqueous solutions of Polyox 301 were fed from a reservoir to the bottom of a packed bed filled with glass beads. The glass beads had diameters of 3/16 and 9/16 in. The tracer was the sodium salt of fluorescein ($Na_2C_{20}H_{10}O_5$, di-sodium salt of 9-0-carboxyphenyl-6-hydroxy-3-isoxanthene). The tracer transmitted light of wavelength in the range of 5,000 to 8,000 Å. A linear relationship exists between the concentration and the intensity of light transmitted by the tracer. Two sets of detecting apparatus (including a phototube, a recorder and a high voltage power supply) were mounted at the bottom and the top of the packed section to detect the input and output concentration of the tracer. Tracer was injected into the calming section very

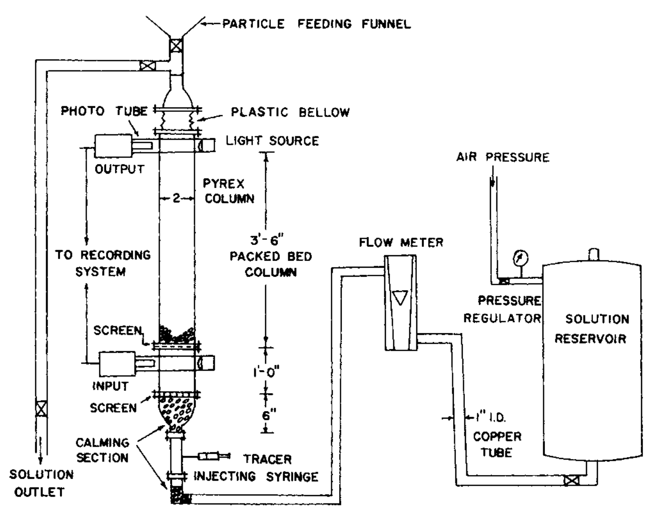

Fig. 1. Schematic diagram of the equipment for pulse response technique on packed bed system.

quickly to generate a sharp pulse. The flow index n and K of power-law fluid were obtained from the measurement of pressure drop and flow rate in tubes.

DISCUSSION

A dispersion model may be represented by

$$\frac{\partial C}{\partial t} = \overline{E}\frac{\partial^2 C}{\partial z^2} - \overline{u}\frac{\partial C}{\partial z} \tag{1}$$

The boundary conditions of Equation (1) are (5)

(a) $\overline{u}C_i = \overline{u}C_{z=0^+} - \overline{E}\frac{\partial C}{\partial z}\bigg|_{z=0^+}$

(b) $\frac{\partial C}{\partial z} = 0$ at $z = L$ for all t

(c) $C = 0$ at $t = 0$

Solving equation (1) and using the moment analysis presented by Aris (*1*) we get

$$\frac{\Delta\sigma^2}{(\Delta\mu)^2} = \frac{2\overline{E}}{\overline{u}L}$$

where $\Delta\mu$ = (first moment of output) − (first moment of input)

$$= \mu_o - \mu_i = \frac{\int_0^\infty tC_o(t)dt}{\int_0^\infty C_o(t)dt} - \frac{\int_0^\infty tC_i(t)dt}{\int_0^\infty C_i(t)dt}$$

$$\Delta\sigma^2 = (\text{variance of output}) - (\text{variance of input})$$

$$= (\sigma_o^2) - (\sigma_i^2)$$

$$= \left\{\frac{\int_0^\infty t^2C_o(t)dt}{\int_0^\infty C_o(t)dt} - \mu_o^2\right\} - \left\{\frac{\int_0^\infty t^2C_i(t)dt}{\int_0^\infty C_i(t)dt} - \mu_i^2\right\}$$

From the experimental pulse data of input and output, the dispersion coefficient $\overline{E}$ can be obtained. The behavior of non-Newtonian fluid (polyoxy solution) used in this work may be approximated by a power-law fluid and is described by $\tau = K(\dot{\gamma})^n$.

In a study of mechanism of liquid phase fluidization with non-Newtonian solutions, Yu et al. (9) obtained the pressure drop correlation, the minimum fluidization velocity, and the bed expansion for power-law fluid through a multiparticle system.

They employed a modified Reynolds number for power-law fluid defined as

$$N_{Re} = \frac{d_p^n\, u_o^{2-n}}{K'8^{n'-1}} \tag{2}$$

$$K' = K\left(\frac{3n+1}{4n}\right)^n$$

The Reynolds number of this study ranged from 7 to 800. Figure 2 shows the data obtained from this work and the correlation line obtained by Chung and Wen (*2, 3*). It is seen that for n greater than 0.81, no substantial difference exists between the $\epsilon \cdot N_{Pe}$ of Newtonian and non-Newtonian fluids in the packed bed based on the Reynolds number defined according to Equation (2). Future investigation in the range of n less than 0.81 is suggested.

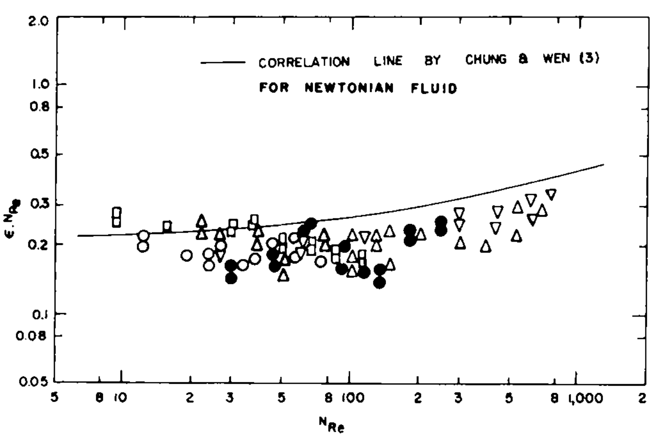

Fig. 2. Correlation of longitudinal dispersion coefficients of non-Newtonian (polyox) solution in packed beds in terms of Peclet number.

		d_p, in.	n, —	K, lb.$_m$/(ft.)(sec.)	ϵ, —
water	▽	9/16	1.0	0.00066	0.5
	●	3/16	1.0	0.00066	0.4
0.1% Polyox solution	△	9/16			0.5
	○	3/16	0.9	0.002	0.4
0.35% Polyox solution	□	9/16	0.81	0.015	0.5

NOTATION

C = concentration, mole L^{-3}
C_i = concentration of input, moles L^{-3}
C_o = concentration of output, moles L^{-3}
d_p = particle diameter, L
$\overline{E}$ = axial dispersion coefficient based on interstitial velocity, $L^2\theta^{-1}$
K = fluid consistency index, $ML^{-1}\theta^{n-2}$
L = reactor length, L
n = flow behavior index, dimensionless
N_{Pe} = Peclet number, $\frac{d_p\overline{u}}{\overline{E}}$, dimensionless
$\overline{u}$ = interstitial velocity, $L\theta^{-1}$
u_o = superficial velocity, $L\theta^{-1}$
z = longitudinal direction, L
$\dot{\gamma}$ = shear rate, θ^{-1}
ϵ = voidage, dimensionless
t = time, θ
τ = sheer stress, $ML\theta^{-2}$

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LITERATURE CITED

1. Aris, R., *Chem. Eng. Sci.*, **9**, 250 (1959).
2. Chung, S. F., Ph.D. dissertation, West Virginia Univ., Morgantown (1968).
3. Chung, S. F. and C. Y. Wen, *AIChE J.*, **14**, 857 (1968).
4. Clements, W. C., Jr., and K. B. Schnelle, Jr., *Ind. Eng. Chem. Process Design Develop.*, **2**, 94 (1963).
5. Danckwerts, P. V., *Chem. Eng. Sci.*, **1**, 1, (1953).
6. Hougen, J. C., and R. A. Walsh, *Chem. Eng. Progr.*, **57**, 69 (1961).
7. Johnson, J. L., Ph.D. dissertation, Kansas State Univ., Manhattan (1966).
8. Justice, R. G., Ph.D. dissertation, Vanderbilt Univ., Nashville, Tenn. (1964).
9. Yu, Y. H., C. Y. Wen, and R. C. Bailie, *Can. J. Chem. Eng.*, **46**, 149 (1968).